Please check the examination details below before entering your candidate information

| Candidate surname | Other names |
|---|---|
| | |

**Pearson Edexcel**
**International Advanced Level**

| Centre Number | Candidate Number |
|---|---|
| | |

**Time** 1 hour 30 minutes

| **Paper reference** | **WMA14/01** |
|---|---|

# Mathematics

## International Advanced Level
## Pure Mathematics P4

| **You must have:** Mathematical Formulae and Statistical Tables (Yellow), calculator | Total Marks |
|---|---|
| | |

**Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.**

## Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

## Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

## Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.
- Good luck with your examination.

*Turn over* ▶

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**1.** Given that $k$ is a constant and the binomial expansion of

$$\sqrt{1+kx} \qquad |kx| < 1$$

in ascending powers of $x$ up to the term in $x^3$ is

$$1 + \frac{1}{8}x + Ax^2 + Bx^3$$

(a) (i) find the value of $k$,

(ii) find the value of the constant $A$ and the constant $B$.

**(5)**

(b) Use the expansion to find an approximate value to $\sqrt{1.15}$

Show your working and give your answer to 6 decimal places.

**(2)**

(a) Get the formula for the Binomial Expansion from the formula booklet:

$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \ldots$, Range of validity $|x|<1$

Substitute into the formula:

$(1+kx)^{\frac{1}{2}} = 1 + (\frac{1}{2})(kx) + \frac{(\frac{1}{2})(-\frac{1}{2})}{2!}(kx)^2 + \frac{(\frac{1}{2})(-\frac{1}{2})(-\frac{3}{2})}{3!}(kx)^3 + \ldots$

$= 1 + \frac{1}{2}kx - \frac{1}{8}k^2x^2 + \frac{1}{16}k^3x^3 + \ldots$

i. Use the coefficient of $x$ given:

$\frac{1}{8} = \frac{1}{2}k$

$\Rightarrow k = \frac{1}{4}$ M1A1

ii. Use the found value of $k$ to get $A$ and $B$:

$A = -\frac{1}{8}(\frac{1}{4})^2 = -\frac{1}{128}$

$B = \frac{1}{16}(\frac{1}{4})^3 = \frac{1}{1024}$

$A = -\frac{1}{128}$, $B = \frac{1}{1024}$ M1A1A1

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**Question 1 continued**

(b) Equate $\sqrt{1.15} = \sqrt{1+\frac{1}{4}x}$ to get the value of $x$ we have to substitute:

$$\sqrt{1.15} = \sqrt{1+\frac{1}{4}x}$$

$$1.15 = 1 + 0.25x$$

$$0.15 = 0.25x$$

$$x = 0.6$$

Substitute $x = 0.6$ into our expansion:

$$\sqrt{1+\frac{1}{4}x} = 1 + \frac{1}{8}x - \frac{1}{128}x^2 + \frac{1}{1024}x^3 + \dots$$

$$\sqrt{1+\frac{1}{4}(0.6)} = 1 + \frac{1}{8}(0.6) - \frac{1}{128}(0.6)^2 + \frac{1}{1024}(0.6)^3 + \dots$$

$$\sqrt{1.15} = 1.072398$$ M1A1





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**Question 1 continued**

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**Question 1 continued**

Q1

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**2.**

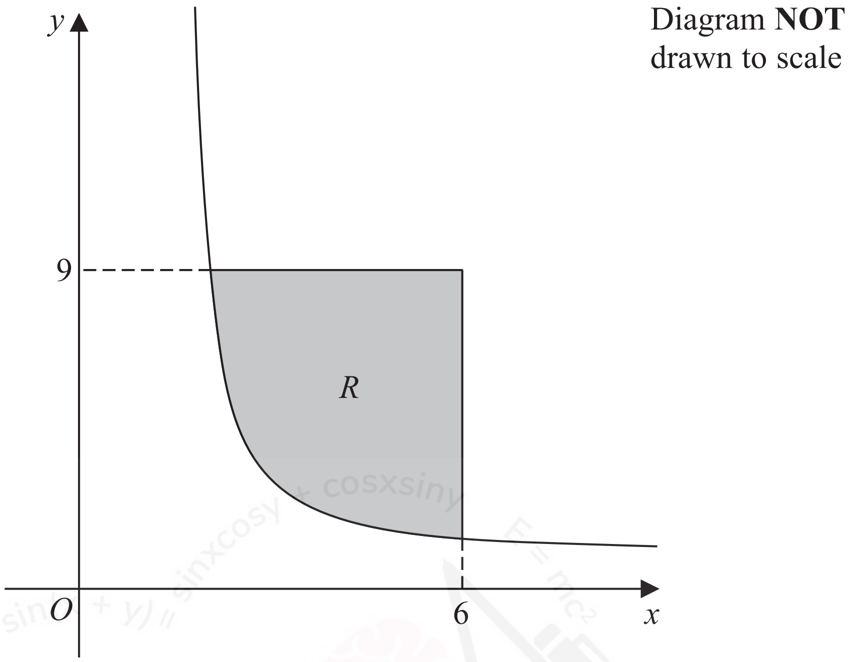



**Figure 1**

Figure 1 shows a sketch of part of the curve with equation

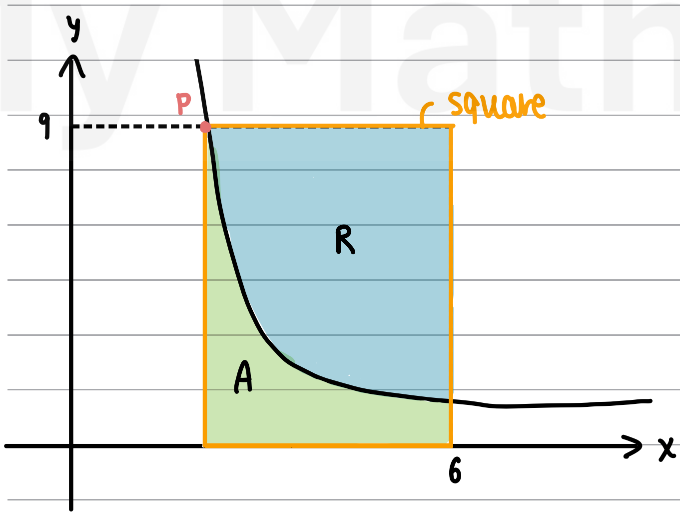
$$y = \frac{9}{(2x-3)^{1.25}} \qquad x > \frac{3}{2}$$

The finite region $R$, shown shaded in Figure 1, is bounded by the curve, the line with equation $y = 9$ and the line with equation $x = 6$

This region is rotated through $2\pi$ radians about the $x$-axis to form a solid of revolution.

Find, by algebraic integration, the exact volume of the solid generated.

**(7)**




We will calculate the volume of the cylinder formed when rotating the square around the x-axis and will then subtract the volume created by rotating A around the x-axis to be left with volume of R.





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**Question 2 continued**

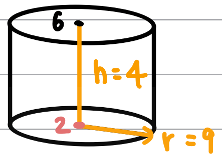
Let's get point P, the lower bound of our integration:

$$9 = \frac{9}{(2x-3)^{1.25}}$$

$$(2x-3)^{1.25} = 1$$

$$2x = 1+3 \Rightarrow x=2 \text{ at } P$$

Get volume of cylinder: B1




$$V = \pi r^2 h$$

$$V_{cyl} = \pi (9)^2 \times 4$$

$$= 324\pi$$

Now let's get volume of A:

Formula for Volume of Revolution around the x-axis:

$$V = \pi \int_a^b y^2 \, dx$$

where a and b are the points on the x-axis bounding the area

$$V_A = \pi \int_2^6 \left( \frac{9}{(2x-3)^{1.25}} \right)^2 dx$$

$$= \pi \int_2^6 \frac{81}{(2x-3)^{2.5}} dx \qquad \text{LB is point } P_x = 2$$

$$= 81\pi \int_2^6 (2x-3)^{-2.5} dx$$

M1A1 $$= 81\pi \left[ \frac{1}{-1.5 \times 2} (2x-3)^{-1.5} \right]_2^6 \qquad \text{Integrate}$$

Reverse chain rule: divide by derivative of bracket

$$= 81\pi \left( -\frac{1}{3}(2(6)-3)^{-1.5} + \frac{1}{3}(2(2)-3)^{-1.5} \right)$$

$$= 81\pi \left( -\frac{1}{81} + \frac{1}{3} \right) \quad \text{dM1}$$

$$= 26\pi \quad \text{A1}$$

Hence to get volume R:

$$V_{cyl} - V_A = V_R \Rightarrow 324\pi - 26\pi = 298\pi \quad \text{ddM1}$$

Volume $R = 298\pi$ A1

Q2

**(Total 7 marks)**



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**3.**

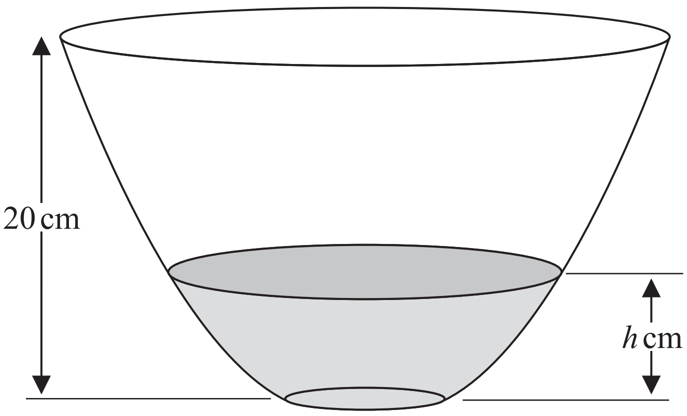


**Figure 2**

A bowl with circular cross section and height 20 cm is shown in Figure 2.

The bowl is initially empty and water starts flowing into the bowl.

$t=0, V=0, h=0$

When the depth of water is $h$ cm, the volume of water in the bowl, $V$ cm$^3$, is modelled by the equation

$$V = \frac{1}{3}h^2(h+4) \qquad 0 \leqslant h \leqslant 20$$

Given that the water flows into the bowl at a constant rate of 160 cm$^3$ s$^{-1}$, find, according to the model,

(a) the time taken to fill the bowl, **(2)**

(b) the rate of change of the depth of the water, in cm s$^{-1}$, when $h = 5$ **(5)**

(a) We want to find $t$ when $h = 20$ (maximum)

We are given that $\frac{dV}{dt} = 160\text{cm}^3\text{s}^{-1}$

Let's get $V$ when $h=20$ with given equation:

$V = \frac{1}{3}(20)^2(24)$

$V = 3200\text{cm}^3$.

Since $160\text{cm}^3$ flow in per second:

$\frac{3200}{160} = 20$ seconds.

$t = 20$s M1A1

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**Question 3 continued**

(b) We want $\frac{dh}{dt}$ at $h=5$.

Let's get $\frac{dV}{dh}$ from the given equation:

$$V = \frac{1}{3}h^2(h+4)$$

$$V = \frac{1}{3}h^3 + \frac{4}{3}h^2$$

$$\frac{dV}{dh} = h^2 + \frac{8}{3}h \quad \text{M1}$$

given found

To get $\frac{dh}{dt}$:

$$\frac{dh}{dt} = \frac{dV}{dt} \times \frac{dh}{dV} \quad \text{M1}$$

$$\frac{dh}{dt} = 160 \times \frac{1}{h^2 + \frac{8}{3}h} \quad \text{A1}$$

$$\left.\frac{dh}{dt}\right|_{h=5} = \frac{160}{5^2 + \frac{8}{3}(5)} = \frac{96}{93} = 4.2\text{cm s}^{-1}$$

$\frac{dh}{dt}$ at $h=5$

dM1 A1



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**Question 3 continued**

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**Question 3 continued**

Q3

**(Total 7 marks)**



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4. Use algebraic integration and the substitution $u = \sqrt{x}$ to find the exact value of

$$\int_1^4 \frac{10}{5x + 2x\sqrt{x}}\,dx$$

Write your answer in the form $4\ln\left(\frac{a}{b}\right)$, where $a$ and $b$ are integers to be found.

(*Solutions relying entirely on calculator technology are not acceptable.*)

**(8)**

To use the given substitution $u = \sqrt{x}$, get new limits and $dx = \ldots$

New limits

| $x$ | | $u$ |
|---|---|---|
| 1 | $u = \sqrt{1} \rightarrow$ | 1 |
| 4 | $u = \sqrt{4} \rightarrow$ | 2 |

To get $dx = \ldots$, differentiate:

$u = x^{\frac{1}{2}}$

$\frac{du}{dx} = \frac{1}{2}x^{-\frac{1}{2}} = \frac{1}{2\sqrt{x}} = \frac{1}{2u}$

$\therefore\ dx = 2u\,du$ B1

Substitute into the Integral:

$$\int_1^4 \frac{10}{5x + 2x\sqrt{x}}\,dx \qquad u = \sqrt{x},\ u^2 = x$$

$$= \int_1^2 \frac{10}{5u^2 + 2u^2(u)} \times 2u\,du \quad \text{M1A1}$$

$$= \int_1^2 \frac{20\cancel{u}}{5u^{\cancel{2}} + 2u^{\cancel{3}2}}\,du$$

$$= \int_1^2 \frac{20}{5u + 2u^2}\,du$$

$$= \int_1^2 \frac{20}{u(5+2u)}\,du \qquad \text{factorize denominator}$$

We need to do Partial fractions on this:

$$\frac{20}{u(5+2u)} = \frac{A}{u} + \frac{B}{5+2u}$$ Manipulate this to get common denominator

$\Rightarrow 20 = A(5+2u) + B(u)$

Substitute values to get A and B:

$u = 0 \Rightarrow 20 = 5A + B(0) \Rightarrow A = 4$

$u = 1 \Rightarrow 20 = 4(5+2) + B \Rightarrow B = -8$

$$\therefore\ \frac{20}{u(5+2u)} = \frac{4}{u} - \frac{8}{5+2u} \quad \text{dM1A1}$$

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**Question 4 continued**

$$= \int_1^2 \frac{4}{u} - \frac{8}{5+2u}\, du$$ use the P.F. we got

Reverse chain rule: divide by derivative of the bracket

$$= \left[ 4\ln u - \frac{8}{2}\ln(5+2u) \right]_1^2$$ Integrate

ddM1

$$= (4\ln(2) - 4\ln(5+4)) - (\cancel{4\ln 1}^{0} - 4\ln(5+2))$$

$$= 4\ln 2 - 4\ln 9 + 4\ln 7$$ M1

$$= 4\ln(2\times 7) - 4\ln 9$$ apply ln law $\ln a + \ln b = \ln a \times b$

$$= 4\ln\frac{14}{9}$$ A1 apply ln law $\ln a - \ln b = \ln\frac{a}{b}$

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**Question 4 continued**







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**Question 4 continued**

Q4

**(Total 8 marks)**

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**5.** A curve has equation

$$y^2 = y\mathrm{e}^{-2x} - 3x$$

(a) Show that

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2y\mathrm{e}^{-2x} + 3}{\mathrm{e}^{-2x} - 2y}$$

**(4)**

The curve crosses the $y$-axis at the origin and at the point $P$.

The tangent to the curve at the origin and the tangent to the curve at $P$ meet at the point $R$.

(b) Find the coordinates of $R$.

**(5)**

(a) ☆ Implicit differentiation:

When differentiating $y$, multiply by $\frac{dy}{dx}$

$$y^2 = ye^{-2x} - 3x$$

Product Rule: $\frac{d}{dx}(uv) = uv' + u'v$

$$2y\frac{dy}{dx} = \frac{dy}{dx}e^{-2x} - 2ye^{-2x} - 3$$ B1M1A1

$$2ye^{-2x} + 3 = \frac{dy}{dx}e^{-2x} - 2y\frac{dy}{dx}$$ collect all $\frac{dy}{dx}$ terms on one side

$$2ye^{-2x} + 3 = \frac{dy}{dx}(e^{-2x} - 2y)$$ factor out $\frac{dy}{dx}$

$$\frac{dy}{dx} = \frac{2ye^{-2x} + 3}{e^{-2x} - 2y}$$ hence shown

A1

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**Question 5 continued**

(b) We need to get tangents at O and at P.

Get Point P first:

On y-axis, $\therefore x=0$: $y^2 = ye^{-0} + 3(0)$

$y^2 = y$

$y^2 - y = 0$

$y(y-1) = 0 \Rightarrow y = 1$ at P. B1

Hence $P(0,1)$.

Get the $\frac{dy}{dx}$ for each tangent using the given $\frac{dy}{dx} = \frac{2ye^{-2x}+3}{e^{-2x}-2y}$

$\left.\frac{dy}{dx}\right|_{y=0,x=0} = \frac{\cancel{2(0)e^{0}}^{0}+3}{\cancel{e^{0}}^{1}-\cancel{2(0)}^{0}} = 3$ gradient of tangent at origin

$\therefore y = 3x$ equation of $t_{OR}$.

$\left.\frac{dy}{dx}\right|_{y=1,x=0} = \frac{2(1)e^{0}+3}{e^{0}-2(1)} = \frac{5}{-1} = -5$ gradient of tangent at P.

Use formula $y - y_1 = m(x - x_1)$ to get the equation:

$y - 1 = -5(x-0)$

$y = 1 - 5x$ equation of $t_P$

M1A1

Now simultaneously solve $t_{OR}$ and $t_P$ to get the coordinates of R:

$3x = 1 - 5x$

$8x = 1 \Rightarrow x = \frac{1}{8}$, $y = \frac{3}{8}$ dM1

Hence $R\left(\frac{1}{8}, \frac{3}{8}\right)$ A1

Q5

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**6.**

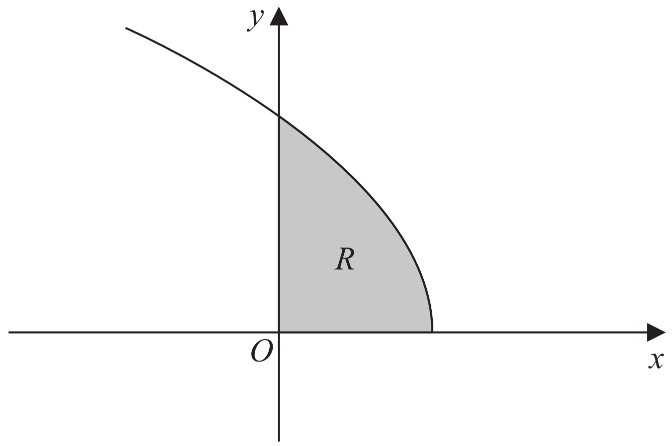


**Figure 3**

Figure 3 shows a sketch of the curve $C$ with parametric equations

$$x = 2\cos 2t \qquad y = 4\sin t \qquad 0 \leqslant t \leqslant \frac{\pi}{2}$$

The region $R$, shown shaded in Figure 3, is bounded by the curve, the $x$-axis and the $y$-axis.

(a) (i) Show, making your working clear, that the area of $R = \int_0^{\frac{\pi}{4}} 32\sin^2 t\cos t \, \mathrm{d}t$

(ii) Hence find, by algebraic integration, the exact value of the area of $R$.

**(6)**

(b) Show that all points on $C$ satisfy $y = \sqrt{ax+b}$, where $a$ and $b$ are constants to be found.

**(3)**

The curve $C$ has equation $y = \mathrm{f}(x)$ where f is the function

$$\mathrm{f}(x) = \sqrt{ax+b} \qquad -2 \leqslant x \leqslant 2$$

and $a$ and $b$ are the constants found in part (b).

(c) State the range of f.

**(1)**

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**Question 6 continued**

(a)i. ★ Parametric Integration

area $= \int_a^b y \frac{dx}{dt} dt$

Let's get limits for t.

Upper Bound: On x-axis, ∴ $y=0$:

$0 = 4\sin t$

$t=0$

Lower Bound: On y-axis, ∴ $x=0$:

$0 = 2\cos 2t$

$t = \frac{\pi}{4}$.

Let's get $\frac{dx}{dt}$:

$x = 2\cos 2t$

$\frac{dx}{dt} = -4\sin 2t$

Substitute into the integral:

$R = \int_{\frac{\pi}{4}}^{0} 4\sin t\,(-4\sin 2t)\,dt$ M1A1

$= \int_{\frac{\pi}{4}}^{0} -16\sin t \sin 2t \; dt$

apply double angle formula

$= \int_{\frac{\pi}{4}}^{0} -16\sin t(2\sin t\cos t)\,dt$ $\sin 2A = 2\sin A\cos A$

$= \int_{\frac{\pi}{4}}^{0} -32\sin^2 t\cos t\;dt$ dM1

$= -32\int_{\frac{\pi}{4}}^{0} \sin^2 t\cos t\;dt$ take constant out

$= 32\int_{0}^{\frac{\pi}{4}} \sin^2 t\cos t\;dt$ multiply Integral by −1 to flip limits

A1 hence shown

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**Question 6 continued**

ii. Integrate what we showed in (a):

$$32\int_0^{\frac{\pi}{4}} \sin^2 t \cos t \, dt$$ Use Integration by Recognition

Form: $\int [f(x)]^n f'(x)\,dx = [f(x)]^{n+1} + c$

$$= 32\left[\frac{1}{3}\sin^3 t\right]_0^{\frac{\pi}{4}}$$

$$= \left(\frac{32}{3}\sin^3\left(\frac{\pi}{4}\right)\right) - \left(\frac{32}{3}\sin^3(0)\right)$$ substitute in limits

M1

$$= \frac{32}{3} - \frac{2\sqrt{2}}{8}$$

area $R = \frac{8\sqrt{2}}{3}$ A1

(b) We need to get the Cartesian equation:

$$x = 2\cos 2t$$ use double angle formula

$$x = 2 - 4\sin^2 t$$ $\cos 2A = 1 - 2\sin^2 A$

Substitute in $\frac{y}{4} = \sin t$:

$$x = 2 - 4\left(\frac{y}{4}\right)^2$$ M1 A1

$$x = 2 - \frac{4y^2}{16}$$

$$x = 2 - \frac{y^2}{4}$$

$$4x = 8 - y^2$$ multiply both sides by 4

$$y^2 = 8 - 4x$$

$$\Rightarrow y = \sqrt{8 - 4x}$$ hence shown

A1

(c) Given domain is $-2 \le x \le 2$:

$$y = \sqrt{8 - 4(-2)}$$

$$= \sqrt{16} = 4$$ upper bound

$$y = \sqrt{8 - 4(2)}$$

$$= \sqrt{0} = 0$$ lower bound

$\therefore$ Range: $0 \le f(x) \le 4$ B1





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**Question 6 continued**

Q6

**(Total 10 marks)**

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**7.** Relative to a fixed origin $O$, the line $l$ has equation

$$\mathbf{r} = \begin{pmatrix} 1 \\ -10 \\ -9 \end{pmatrix} + \lambda \begin{pmatrix} 4 \\ 4 \\ 2 \end{pmatrix} \quad \text{where } \lambda \text{ is a scalar parameter}$$

Given that $\overrightarrow{OA}$ is a unit vector parallel to $l$,

(a) find $\overrightarrow{OA}$

**(2)**

The point $X$ lies on $l$.

Given that $X$ is the point on $l$ that is closest to the origin,

(b) find the coordinates of $X$.

**(5)**

The points $O$, $X$ and $A$ form the triangle $OXA$.

(c) Find the exact area of triangle $OXA$.

**(3)**

(a) Formula for unit vector:

$$\hat{a} = \frac{a}{|a|}$$ vector / magnitude of vector

Formula for Magnitude of a vector:

let $\overrightarrow{AB} = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$

$$|\overrightarrow{AB}| = \sqrt{a^2+b^2+c^2}$$

Use the direction vector of $l$ since $\overrightarrow{OA}$ and $l$ are parallel:

$$\overrightarrow{OA} = \hat{a} = \frac{\begin{pmatrix} 4 \\ 4 \\ 2 \end{pmatrix}}{\sqrt{4^2+4^2+2^2}}$$ M1

$$\overrightarrow{OA} = \frac{\begin{pmatrix} 4 \\ 4 \\ 2 \end{pmatrix}}{6}$$

$$\therefore \overrightarrow{OA} = \begin{pmatrix} \frac{2}{3} \\ \frac{2}{3} \\ \frac{1}{3} \end{pmatrix}$$ A1

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**Question 7 continued**

(b) Since X is closest point to origin, $\overrightarrow{OX}$ must be perpendicular to $l$.

$\therefore \overrightarrow{OX} \cdot (\text{direction vector}) = 0$

Let's take $\overrightarrow{OX}$ to be the general point of $l$:

$$\overrightarrow{OX} = \begin{pmatrix} 1+4\lambda \\ -10+4\lambda \\ -9+2\lambda \end{pmatrix} \quad \text{M1}$$

Formula for dot product:
$\begin{pmatrix} a \\ b \\ c \end{pmatrix} \cdot \begin{pmatrix} d \\ e \\ f \end{pmatrix} = ad + be + cf$

$$\begin{pmatrix} 1+4\lambda \\ -10+4\lambda \\ -9+2\lambda \end{pmatrix} \cdot \begin{pmatrix} 4 \\ 4 \\ 2 \end{pmatrix} = 0$$

$$4(1+4\lambda) + 4(-10+4\lambda) + 2(-9+2\lambda) = 0 \quad \text{dM1}$$

$$4 + 16\lambda - 40 + 16\lambda - 18 + 4\lambda = 0$$

$$36\lambda = 54$$

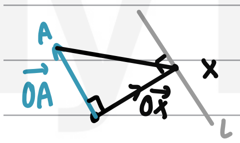
$$\lambda = \frac{3}{2} \quad \text{ddM1 A1}$$

Substitute our found $\lambda$ into $\overrightarrow{OX}$ to get the coordinates of X:

$$\overrightarrow{OX} = \begin{pmatrix} 1+4(1.5) \\ -10+4(1.5) \\ -9+2(1.5) \end{pmatrix} = \begin{pmatrix} 7 \\ -4 \\ -6 \end{pmatrix}$$

$\therefore X(7, -4, -6)$ A1

(c) Diagram




$\overrightarrow{OA}$ and $\overrightarrow{OX}$ are also perpendicular since $\overrightarrow{OA}$ is parallel to line $l$.

We know $|\overrightarrow{OA}| = 1$ since it's a unit vector

Get $|\overrightarrow{OX}|$:

$$|\overrightarrow{OX}| = \sqrt{7^2 + 4^2 + 6^2} \quad \text{from (b) } \overrightarrow{OX} = \begin{pmatrix} 7 \\ -4 \\ -6 \end{pmatrix}$$

$$= \sqrt{101}. \quad \text{M1}$$

Hence area of triangle OXA:

$$\text{area} = \frac{\sqrt{101} \times 1}{2} \quad \text{dM1}$$

$$\therefore \text{area} = \frac{\sqrt{101}}{2} \quad \text{A1}$$

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**Question 7 continued**

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**Question 7 continued**

Q7

**(Total 10 marks)**

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**8.** (a) Given that $y = 1$ at $x = 0$, solve the differential equation

$$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{6xy^{\frac{1}{3}}}{\mathrm{e}^{2x}} \qquad y \geqslant 0$$

giving your answer in the form $y^2 = \mathrm{g}(x)$.

**(7)**

(b) Hence find the equation of the horizontal asymptote to the curve with equation $y^2 = \mathrm{g}(x)$.

**(2)**

(a)

$$\frac{dy}{dx} = \frac{6xy^{\frac{1}{3}}}{e^{2x}}$$

$$\int \frac{1}{6y^{\frac{1}{3}}}\,dy = \int \frac{x}{e^{2x}}\,dx \quad \text{B1}$$

Separate Variables by grouping the like terms on one side

$$\int \frac{1}{6}y^{-\frac{1}{3}}\,dy = \int xe^{-2x}\,dx$$

Standard Integration

multiplication, ∴ By Parts

$$\int uv'\,dx = uv - \int u'v\,dx$$

RHS

$u = x \xrightarrow{\frac{d}{dx}} u' = 1$

$v = -\frac{1}{2}e^{-2x} \xleftarrow{\int} v' = e^{-2x}$

$$\frac{1}{6} \times \frac{3}{2}y^{\frac{2}{3}} = -\frac{1}{2}e^{-2x}x - \int -\frac{1}{2}e^{-2x}\,dx \quad \text{M1A1}$$

$$\frac{1}{4}y^{\frac{2}{3}} = -\frac{1}{2}xe^{-2x} - \frac{1}{4}e^{-2x}$$

multiply both sides by 4

$$y^{\frac{2}{3}} = -2xe^{-2x} - e^{-2x} + c \quad \text{dM1A1}$$

Use the given $y=1, x=0$ to get $+c$:

$$1 = \cancel{-2(0)e^{0}} - \cancel{e^{0}} + c$$

$c = 2$ M1

$$\therefore\ y^{\frac{2}{3}} = -2xe^{-2x} - e^{-2x} + 2$$

cube both sides

$$y^2 = (-2xe^{-2x} - e^{-2x} + 2)^3 \quad \text{A1}$$

(b) To get horizontal asymptote: as $x \to \infty$, $e^{-2x} \to 0$

$$y^2 = \left(\cancel{\frac{-2x}{e^{2x}}}^{\,0} - \cancel{\frac{1}{e^{2x}}}^{\,0} + 2\right)^3 \quad \text{M1}$$

$$y^2 = 2^3$$

$\Rightarrow y = 2\sqrt{2}$ horizontal asymptote A1



P 6 6 0 0 8 A 0 2 6 3 2

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**Question 8 continued**





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**Question 8 continued**

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**Question 8 continued**

Q8

**(Total 9 marks)**





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**9.** (i) Relative to a fixed origin $O$, the points $A$, $B$ and $C$ have position vectors $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$ respectively.

Points $A$, $B$ and $C$ lie in a straight line, with $B$ lying between $A$ and $C$.

Given $AB:AC = 1:3$ show that

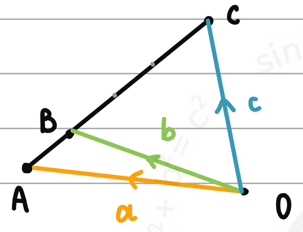
$$\mathbf{c} = 3\mathbf{b} - 2\mathbf{a}$$

**(3)**

(ii) Given that $n \in \mathbb{N}$, prove by contradiction that if $n^2$ is a multiple of 3 then $n$ is a multiple of 3

**(5)**

i. Diagram




$\vec{AB} = \vec{OB} - \vec{OA}$ $\quad$ $\vec{AC} = \vec{OC} - \vec{OA}$

$= b - a$ $\quad$ $= c - a$ M1

Since $\vec{AC} = 3\vec{AB}$:

$c - a = 3(b - a)$ dM1

$c - a = 3b - 3a$

A1 $c = 3b - 2a$ hence shown

ii.

Make an Assumption:

"Assume that there exists a number n such that n is not a multiple of 3, but $n^2$ is a multiple of 3" B1

we want our assumption to be the opposite of what we are trying to prove. In the Proof, we are trying to contradict the assumption

If n is not a multiple of 3:

$n = 3m + 1$

Try to get $n^2$:

$n^2 = (3m+1)^2$

$= 9m^2 + 6m + 1$ M1

$n^2 = 3(3m^2 + 2m) + 1$ is 1 more than a multiple of 3, $\therefore$ not a multiple of 3.

M1 A1 This is a contradiction.

Hence if n is a multiple of 3 then $n^2$ is a multiple of 3 proven by contradiction.

A1

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**Question 9 continued**

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**Question 9 continued**

Q9

**(Total 8 marks)**

**TOTAL FOR PAPER IS 75 MARKS**

**END**